



ZIMBABWE

MINISTRY OF HIGHER AND TERTIARY EDUCATION,  
INNOVATION, SCIENCE AND TECHNOLOGY DEVELOPMENT

HIGHER EDUCATION EXAMINATIONS COUNCIL  
(HEXCO)

HIGHER NATIONAL DIPLOMA

IN

CIVIL ENGINEERING/SURVEYING AND GEOMATICS

SUBJECT: Engineering Mathematics 3

PAPER NO: 778/17/S06

DURATION: 3 hours

**MARCH/APRIL 2022 EXAMINATION**

**REQUIREMENTS**

1. Answer booklet.
2. Scientific calculator.

**INSTRUCTIONS TO CANDIDATE**

1. Answer any four (4) questions.
2. Each question carries 25 marks.
3. Show all workings.
4. Start each question on a fresh page.

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*This paper consists of 3 printed pages.*

QUESTION 1

a) If  $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$

Find two non-singular matrices P and Q such that  $PAQ = I$ . Hence find  $A^{-1}$ . (10 marks)

b) Find the middle term in the expansion of  $(x + y)^{13}$ . (5 marks)

c) Reduce the matrix A to Echelon form:

$A = \begin{pmatrix} 2 & 2 & 4 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{pmatrix}$  (10 marks)

QUESTION 2

a) Find the solution of the differential equation  $\frac{dy}{dx} - x \tan(y - x) = 1$  (4 marks)

b) Solve  $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$  (8 marks)

c) The deflection of a strut of length L with one end ( $x = 0$ ) built in and the other supported and subjected to end thrust P, satisfies the equation  $\frac{d^2y}{dx^2} + a^2y = \frac{a^2R}{P}(L - x)$ .

Prove that the deflection curve is  $y = \frac{R}{P} \left( \frac{1}{a} \sin ax - L \cos ax + L - x \right)$  where  $al = \tan al$ , given that at  $x = 0$ ,  $y = \frac{dy}{dx} = 0$ . (13 marks)

QUESTION 3

a) Solve  $(x^2 + y^2) \frac{dy}{dx} = xy$  (7 marks)

b) Solve  $\frac{dy}{dx} = \frac{-2xy^3 + 2}{3x^2y^2 + 8e^{4y}}$  (5 marks)

c) Using the method of variation of parameters, find the general solution of  $y^{11} + 4y = \sec x$  for  $-\frac{\pi}{4} < x < \frac{\pi}{4}$  (7 marks)

d) Use the method of undetermined coefficients to find the general solution of  $y^{11} - 5y^1 + 6y = -3 \sin 2x$ . (6 marks)

QUESTION 4

- a) From the first principle, find the Laplace transform of  $(1 + \cos 2t)$  (5 marks)
- b) Show that the Laplace transform of  $\cosh at$  is  $\frac{s}{s^2 - a^2}$  (5 marks)
- c) Find the Fourier series expansion of the periodic function of period  $2\pi$ , if  $f(x) = x^2$ ,  $-\pi \leq x \leq \pi$ . Hence, find the sum of the series  $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$   
 Draw the graph of the series in the range  $-3\pi \leq x \leq 3\pi$ . (10 marks)
- d) A periodic function of period 4 is defined as  $f(x) = |x|$ ,  $-2 < x < 2$ . Find its Fourier series expansion. (5 marks)

QUESTION 5

- a) The following data regarding the heights ( $y$ ) and weights ( $x$ ) of 100 college students are given:
- $\sum x = 15\,000$
- $\sum x^2 = 2\,272\,500$
- $\sum y = 6\,800$
- $\sum y^2 = 463\,025$
- $\sum xy = 1\,022\,250$
- i) Find the equation of the regression of height ( $y$ ) on weight ( $x$ ). (5 marks)
- ii) Calculate the correlation coefficient between height and weight and comment. (5 marks)
- b) From the following frequency distribution, compute the standard deviation of the 100 students.

Mass in kg	60 - 62	63 - 65	66 - 68	69 - 71	72 - 74
Frequency	5	18	42	27	8

(7 marks)

- c) Find the eigen value and eigen vectors of the matrix  $A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$  (8 marks)